ON THE KINEMATIC, KINETOSTATIC AND DYNAMIC PROPERTIES OF PARALLEL MANIPULATORS IN THE PRESENCE OF SINGULARITY

SEBASTIEN BRIOT, VIGEN ARAKELIAN
Département de Génie Mécanique et Automatique
National Institute of Applied Sciences (INSA), Rennes, France

Introduction

Over the past decades, with the large development of parallel structures, more attention has been paid to their kinematic, kinetostatic and dynamic properties, and in particular, to their singularities. Several papers deal with singularity analysis of parallel manipulators [1-11]. Most of them present the analysis of singular configurations from a kinematic point of view [1-5]. Algebraically, a singularity analysis is based on the degeneracy of Jacobian matrices of the mechanical structure, or of the system of reciprocal screws (wrenches) applied to the platform by the legs. However, it is also known that, when parallel manipulators have Type 2 singularities [1], they lose their stiffness and their quality of motion transmission, as a result, their payload capability. Therefore, the singularity zones in the workspace of manipulators may be analysed not only in terms of kinematic criterions, from the theoretically perfect model of manipulators, but also in terms of kinetostatic approaches [6-8]. However, the proposed criterions are difficult to use, above all when applied to manipulators involving both translational and rotational degrees of freedom (DOF).

Moreover, while it is demonstrated using the kinetostatic approach that, when subjected to Type 2 singularities parallel manipulator lock up, it has been shown experimentally that, via optimal dynamic control of manipulators, it is possible to pass through these singular zones [9-11]. Thus, singular configurations should also be examined in terms of dynamic aspects.

In this paper, we present an overview of our recent works on the kinematic, kinetostatic and dynamic aspects of singularities. As an application of our works, the singular configurations of a new manipulator developed at the INSA of Rennes (PAMINSA: **PA**rallel **M**anipulator of the **INSA** [12]) are studied from kinematic, kinetostatic and dynamic points of

Sébastien Briot, Vigen Arakelian, 20 avenue des buttes de Coësmes, CS 14315, 35043 Rennes Cedex – France, Phone : +33 2 23 23 84 92, Fax : +33 2 23 23 87 26, vigen.arakelyan@insa-rennes.fr

view. This paper is divided up as follows. The following section presents the kinematic analysis of singularity in more detail, and a brief review of previous works on this topic is carried out. Then, a kinetostatic analysis is performed. For this purpose, the pressure angle is used as a kinetostatic indicator of force transmission. Finally, we present the optimal dynamic conditions generating stable motions inside the singular zones. To obtain a demonstration of the mentioned properties, experimentations are performed on a prototype of PAMINSA.

**Kinematic analysis of singular configurations**

The singularities are defined in [1] as configurations of manipulators where the platform may instantaneously lose (Type 1 singularities) or gain (Type 2 singularities or also constraint singularities [3]) one (or more) DOF. From the kinematic point of view, several methods can be used in order to find the singularity loci of a manipulator, or to interpret the kinematic meaning of such configurations. These methods can be classified into two different classes:

- algebraic approaches, such as the study of the degeneracy of the Jacobian matrices of the mechanical structure, or of the system of reciprocal screws (wrenches) applied to the platform by the legs [1-3].
- geometric approaches, such as analyses using screw theory, Grassman geometry [4, 5];

Each of these methods enable us to find, symbolically or numerically, the singularity loci of parallel mechanisms and to analyse the behaviour of the manipulator in such configurations. However, they are able to define the exact loci of singularities only when taking into account the perfect theoretical model of the mechanism and do not enable us to analyze the loss of functionality of the manipulator in the domain of singular configurations. For such a study, it is necessary to carry out a kinetostatic analysis of the singular configuration. This will be developed in the next section. Firstly, we would like to present an example of the application of the kinematic analysis of singularities on the PAMINSA manipulator (Fig. 1).
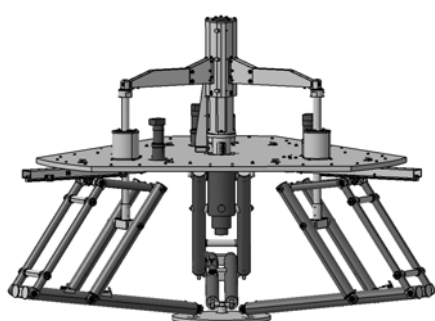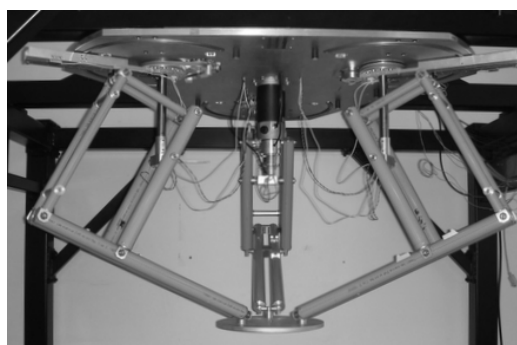
(a) CAD view   (b) prototype
Fig. 1. The PAMINSA manipulator.

The PAMINSA is a manipulator which performs Shoenflies motions. It belongs to a new family of partially decoupled parallel robots having between three and six DOF which was recently developed at the INSA of Rennes (France). The particularity of the PAMINSA is the decoupling of the displacements of the platform in the horizontal plane from the translations along the vertical axis. In order to understand the remainder of the paper, it is of utmost importance to take into account that, due to its decoupling properties, the kinematic model for the planar displacement of the PAMINSA is equivalent to a planar parallel 3-*RPR* robot, with the first revolute joints actuated. For further information, we advise the reader to refer to [12].

In [12-14], the singular configurations of the PAMINSA are analysed in terms of kinematic aspects, via both geometric and algebraic approaches. In these papers, it is shown that Type 1 singularities appear when:

- points A_i, B_i and C_i of manipulator's legs are aligned along a vertical axis (Fig. 2a); in such a case, the leg actuated in rotation about the vertical axis does not produce a displacement of the platform;
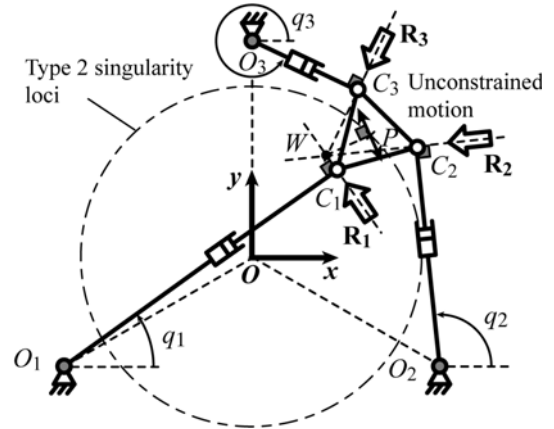- the parallelogram $A_iF_iD_iE_i$ degenerates into a line.

The Type 2 singular configurations appear when:
- the orientation angle ϕ of the platform around the vertical axis is equal to $\pm\cos^{-1}(PC_i/OO_i)$, P being the centre of the platform and O the origin of the base frame ($C_1C_2C_3$ and $O_1O_2O_3$ representing equilateral triangles, P and O are the centre of the circumcircle of these triangles, respectively); in such a case, the platform gains an infinitesimal rotation;
- point P of the platform is located on a circle centred in O, of which the radius is equal to $\sqrt{PC_i^2+OO_i^2-2\,PC_i\,OO_i\cos\phi}$; in such a case, the platform gains a rotation around a vertical axis passing through point W (Fig. 2b) and perform a Cardanic self motion [14].

Let us now deal with the kinetostatic aspects of singular configurations.
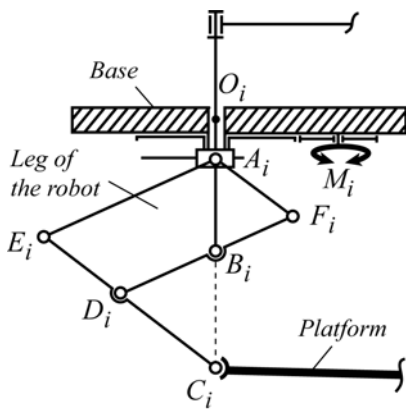
(a) Type 1 singular configuration   (b) Type 2 singular configuration (planar equivalent model)

Fig. 2. Example of singularities of PAMINSA manipulator.

**Kinetostatic analysis of singular configurations**

It is well known that, when parallel manipulators are close to Type 2 singularities, they lose their stiffness along with their quality of motion transmission, and as a result, their payload capability. Therefore, the singularity zones in the workspace of manipulators cannot be defined by kinematic criterions only, from the theoretical perfect model of manipulators, but must also take into account kinetostatic approaches.

The most common indices used to characterize the loss of the quality of motion transmission are conditioning or dexterity indices [6-8]. However, as pointed out in [7], the major drawbacks of these concepts are that they mix arbitrary translational and rotational capabilities and that they usually do not enable a variation in the choice of units. Therefore, we propose using another index, which, contrary to the previous ones, is invariant and has a strong physical meaning: the pressure angle [15, 16]. It will be used as a kinetostatic indicator of force transmission and may characterize the "distance" of the mechanism from the Type 2 singularity.

For a parallel mechanism with n legs, the pressure angle is defined as the angle between the force **F** applied by the leg i (i = 1 to n) to the platform at point C_i and the passive velocity of the platform at the same point, when the actuator of leg i is disconnected. Therefore, for such a mechanism, n pressure angles exist. It could be proved that, when the platform is

subject to Type 2 singularities, the value of the pressure angle is equal to 90°. On the contrary, when the manipulator is at an isotropic configuration, the pressure angle is equal to 0°. Therefore, it is preferable to have a mechanism of which the maximal values of the pressure angles inside its workspace is as close as possible to 0°.

Let us examine the pressure angle of the PAMINSA. In the absence of a gravity field, and due to its decoupling properties, its pressure angle is completely equivalent to that of the 3-$\underline{R}PR$ mechanism. Thus, the pressure angle α_1 of leg 1 is defined as the angle between the force $\mathbf{R_1}$ applied by the leg 1 to the platform and the passive velocity of the platform $\mathbf{V_{C1}}$ when leg 1 is disconnected (Fig. 3a). For more detailed explanations on the calculation of angle α_1, please refer to [16]. As the mechanism has three legs, three pressure angles (α_1, α_2, α_3) can be derived by the same approach. The kinetostatic criterion we propose to use is $\alpha_m = \max(\alpha_1, \alpha_2, \alpha_3)$ (Fig. 3b).

(a) definition   (b) α_m in the workspace ($\phi=0°$)
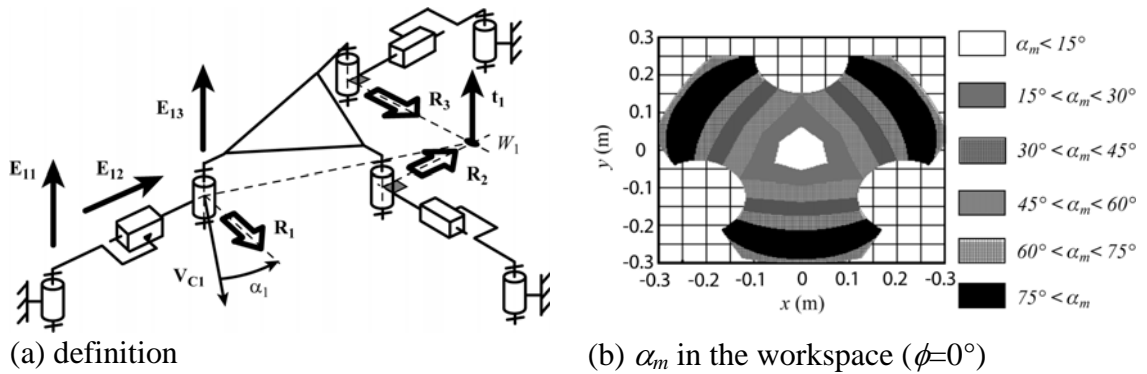
Fig. 3. The pressure angle of the planar parallel 3-$\underline{R}PR$ mechanism.

In Fig. 3b, the black zones are the surfaces where the pressure angle has inadmissible values, and as a result these are the zones in which the parallel mechanism may block. However, it is possible to mention that the locking of the mechanism may be avoided if the system of wrenches applied to the mechanism complies with the condition presented in the next section.

Dynamic analysis of singular configurations

While it is demonstrated using the kinetostatic approach that subject to Type 2 singularities a parallel manipulator locks up, it has been shown experimentally that via optimal dynamic control of manipulators it is possible to pass through these singular zones [9-11]. Thus, singular configurations should also be examined in terms of dynamic aspects.

In [17], we have determined the optimal dynamic conditions generating stable motions inside the singular zones. The obtained results show that the general conditions for passing through singularities can be defined as: the end-effector being able to pass through the singularity without perturbation of motion if the wrench applied to the end-effector by the legs and external efforts is orthogonal to the twist along the direction of the uncontrollable motion. This condition is obtained via inverse dynamics and derived in the example of the PAMINSA manipulator.

For validating this condition, we have carried out experimental tests on the prototype of PAMINSA (Fig. 1b). We have applied an arbitrary fifth order control law and observed the reproduction of motion during the displacement of the platform. The obtained trajectory is shown in Fig. 4 (dashed line). The circle in dotted line corresponds to the singularity loci. The different positions are classified in terms of time. For positions from (a) to (d), the platform moves towards the singular zone yet remains outside of it. In this case, the reproduction of the real trajectory is similar to that sought after. At position (e), the manipulator enters the singular zone, which is close to the circle of the theoretical singular loci, and starts an

uncontrollable motion. Thus, since the motion generation is carried out by non optimized dynamic parameters, the platform moves along an unplanned trajectory (see positions (e) - (h) in Fig. 8).

Next, we have implemented a trajectory which makes it possible to comply with the previous condition and have observed the behaviour of the platform during the displacement. While not shown here, during each displacement, the manipulator retains its orientation constant and passes through the singular configuration without any perturbation. For more detailed information, the reader should refer to [17].
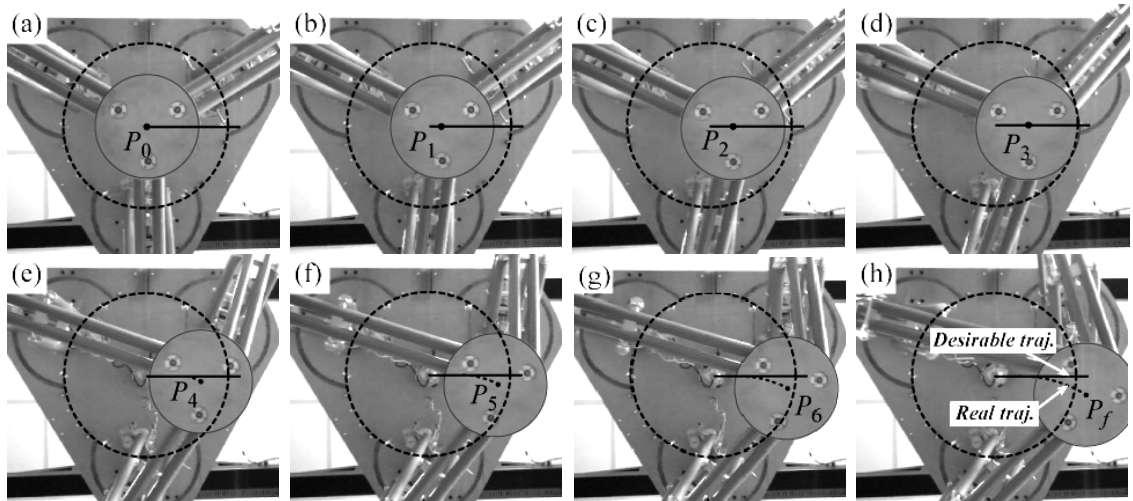
Fig. 4. Trajectory reproduction on the PAMINSA during the displacement of the platform with an arbitrary fifth order polynomial law (view from below).

Conclusions

In this paper, we have presented an overview of our recent works on the kinematic, kinetostatic and dynamic aspects of singularities. As an application of our works, the singular configurations of a new manipulator developed at the INSA of Rennes, named PAMINSA, are studied from kinematic, kinetostatic and dynamic points of view. The singular configurations are firstly derived from kinematic aspects. Then, a kinetostatic analysis is performed. For this purpose, the pressure angle is used as a kinetostatic indicator of force transmission. Finally, we present the optimal dynamic conditions generating stable motions inside the singular zones. As a demonstration of the mentioned properties, experimentations are performed on a prototype of PAMINSA.

REFERENCES

[1] Gosselin, C.M., Angeles, J.: Singularity Analysis of Closed-Loop Kinematic Chains. IEEE Transactions on Robotics and Automatics, Vol.6, No.3, pp.281-290, 1990.

[2] Zlatanov, D., Fenton, R.G., Benhabib, B.: Singularity Analysis of Mechanisms and Robots via a Velocity-Equation Model of the Instantaneous Kinematics. Proceedings of the 1994 IEEE International Conference on Robotics and Automation, Vol.2, pp.980-991, 1994.

[3] Zlatanov, D., Bonev, I.A., Gosselin, C. M.: Constraint Singularities of Parallel Mechanisms. IEEE International Conference on Robotics and Automation (ICRA 2002), Washington, D.C., USA, May 11-15, 2002.

[4] Merlet, J.-P.: Singular Configurations of Parallel Manipulators and Grassmann Geometry. The international Journal of Robotics Research. Vol.8, No.5, pp.45-56, 1989.

[5] Bonev, I.A., Zlatanov, D., Gosselin, C.M.: Singularity Analysis of 3-DOF Planar Parallel Mechanisms via Screw Theory. ASME Journal of Mechanical Design, Vol.125, No.3, pp.573–581, 2003.

[6] Gosselin, C.M.: The Optimum Design of Robotic Manipulators using Dexterity Indices. Robotics and Autonomous Systems, Vol.9, No.4, pp.213–226, 1992.

[7] Merlet, J.-P.: Jacobian, Manipulability, Condition Number, and Accuracy of Parallel Robots. ASME Journal of Mechanical Design, Vol.128, No.1, pp.199–206, 2006.

[8] Angeles, J.: Fundamentals of Robotic Mechanical Systems: Theory, Methods, and Algorithms, 3rd edition, Springer, New York, 2007.

[9] Nenchev, D.N., Bhattacharya, S., Uchiyama, M.: Dynamic Analysis of Parallel Manipulator under the Singularity-Consistent Parameterization. Robotica, Vol.4, No.4, pp.375-384, 1997.

[10] Kemal Ider., S.: Inverse Dynamics of Parallel Manipulators in the Presence of Drive Singularities. Mechanism and Machine Theory, Vol.40, pp.33-44, 2005.

[11] Maass, J., Kolbus, M., Budde, C., Hesselbach, J., Schumacher, W.: Control Strategies for Elarging a Spatial Robot's Workspace by Change of Configuration. Proceedings of the 5th Chemnitz Parallel Kinematics Seminar, pp.515-530, Chemnitz, Germany, 2006.

[12] Briot, S., Arakelian, V., Guegan, S.: PAMINSA: a New Family of Partially Decoupled Parallel Manipulators," Mechanism and Machine Theory, in press.

[13] Arakelian, V., Briot, S., Glazunov, V.: Singular Position of a PAMINSA Parallel Manipulator. Journal of Machinery, Manufacture and Reliability, Allerton Press Inc., No.1, pp.62-69, 2006.

[14] Briot, S., Bonev, I.A., Chablat, D., Wenger, P., Arakelian, V.: Self Motions of General 3-RPR Planar Parallel Robots. The International Journal of Robotics Research, Sage Publications, in press.

[15] Balli, S., Chand, S.: Transmission Angle in Mechanisms. Mechanism and Machine Theory, Vol.37, pp.175-195, 2002.

[16] Arakelian, V., Briot, S., Glazunov, V.: Increase of Singularity-Free Zones in the Workspace of Parallel Manipulators Using Mechanisms of Variable Structure. Mechanism and Machine Theory, in press.

[17] Briot, S., Arakelian, V.: On the Dynamic Properties and Optimum Control of Parallel Manipulators in the Presence of Singularity. Proceedings of the 2008 IEEE International Conference on Robotics and Automation (ICRA 2008), Pasadena, Californie, USA, May 19-23, 2008.

ON THE KINEMATIC, KINETOSTATIC AND DYNAMIC PROPERTIES OF PARALLEL MANIPULATORS IN THE PRESENCE OF SINGULARITY

SUMMARY

This paper presents the singularity analysis of parallel manipulators from kinematic, kinetostatic and dynamic points of view. Algebraically, a singularity analysis is based on the properties of the Jacobian matrices of the mechanical structure, i.e. when the matrices relating the input and output speeds become rank deficient. However, when parallel manipulators are close to presenting Type 2 singularities, they lose their stiffness and their quality of motion transmission. Therefore, the singularity zones may also be defined via kinetostatic approaches. For this purpose, the pressure angle is used as a kinetostatic indicator of force transmission. However, while it is demonstrated that in Type 2 singularities a parallel manipulator locks up, it has been shown experimentally that, via optimal dynamic control of manipulators, it is possible to pass through these singular zones. Hence, we have determined the optimal dynamic conditions generating stable motions inside the singular zones from the inverse dynamic equations.